CALIFORNIA STATE UNIVERSITY
1949
LONG BEACH

Lab#2: Analysis of Free Jet Flow

CE 336

Department of CECEM

Group Report

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Purpose of Study

The purpose of this experiment is to apply Bernoulli's principle as well as mass conservation to measure free jet flow through an orifice. Using the principles we will determine the coefficient of contraction, coefficient of velocity, and coefficient of discharge of the orifice. Bournoulli's principle will also be applied to measure time to empty a tank.

Introduction

In this experiment, water was flowing as a free jet through an orifice with constant circular cross sectional area with different water levels in a controlled tank. The inflow rate from the control valve was matched with the outflow rate by the orifice to create a steady system. From there, the vertical and horizontal displacements as well as the vena-contracta were measured per total head. Each total head was also redirected to fill a large bucket to 50 lbs. and the time in doing so was recorded. The final phase of the experiment was to time how long it took for the orifice to empty the tank of water beginning at 50 inches of total head down to 30 inches with intervals of 5 inches. This data collected was then used to determine the coefficient of contraction (C_c), coefficient of velocity (C_v), and coefficient of discharge (C_d) of the orifice.

Theory

Determining coefficient of contraction, velocity, and discharge

Coefficient of Contraction: $A_c = C_c A_o$ (1)

A_c is the cross-sectional area of the jet flow, A_o is the cross-sectional area of the orifice, and C_c is the coefficient of contraction ($C_c < 1$) and theoretically $C_c = \frac{\Pi}{\Pi+2} = 0.611$.

Coefficient of Velocity: $V_a = C_v V_t$ (2)

Theoretical velocity V_t can be solved for using Bernoulli's equation along the streamline with the static head h, using the equation:

$V_t = \sqrt{2gh}$ (3)

C_v is the coefficient of velocity and is usually very close to 1 and ranges ~ 0.96=0.99. Actual flow rate Q_a is a result of the actual velocity at the vena contracta and the area of the jet at the vena contracta:

$Q_a = V_a A_c = C_v\sqrt{2gh}$ x $C_c A_o = C_v C_c A_o \sqrt{2gh}$ (4)

C_v and C_c can be put together to form coefficient of discharge C_d such that $C_d = C_c C_v$ and that leads to equation (4) being written as:

$Q_a = C_d A_o \sqrt{2gh}$ (5)

To find the coefficient of velocity theoretically is difficult, so the way to approach it is by knowing the x and y coordinates of the free jet leaving the orifice.

$V_a = \frac{x}{t}$ (6)

$y = \frac{gt^2}{2}$ (7)

$C_v = \sqrt{\frac{x^2}{4yh}} = \frac{x}{2\sqrt{yh}}$ (8)

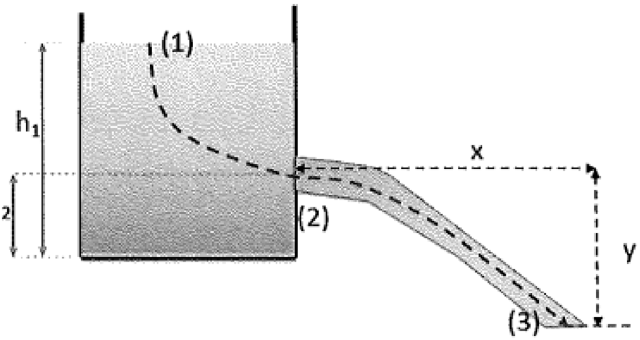


Figure 1: Free jet flowing through an orifice meter

Time to empty a tank

$$\frac{dm}{dt} = \dot{m}_{in} - \dot{m}_{out} = 0 - \dot{m}_{out}$$

$$\Rightarrow \rho \frac{d\forall}{dt} = -\rho Q_a$$

$$\Rightarrow A_T \frac{dh}{dt} = -C_d A_o V_t = -C_d A_o \sqrt{2gh}$$

$$\Rightarrow \frac{dh}{\sqrt{h}} = -\frac{C_d A_o}{A_T} \sqrt{2g}\, dt$$

$$\Rightarrow \int_{h_1}^{h_2} \frac{dh}{\sqrt{h}} = -\frac{C_d A_o}{A_T} \sqrt{2g} \int_o^t dt$$

$$t = \frac{2A_t}{C_d A_o \sqrt{2g}} [\sqrt{h_1} - \sqrt{h_2}] \quad (9)$$

$\dot{m}_{in}$ and $\dot{m}_{out}$ are the incoming and outgoing mass flow rate from the tank with the cross sectional area A_t.

Equipment

Figure 2: Water Tanks

Figure 3: Tank Manometer

Figure 4: Floor Pan

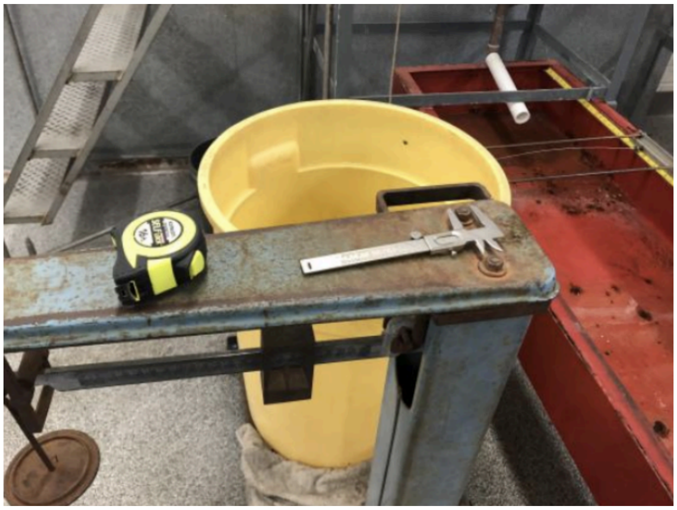


Figure 5: Measuring Tape, Caliper, and Bucket

Figure 6: Orifice

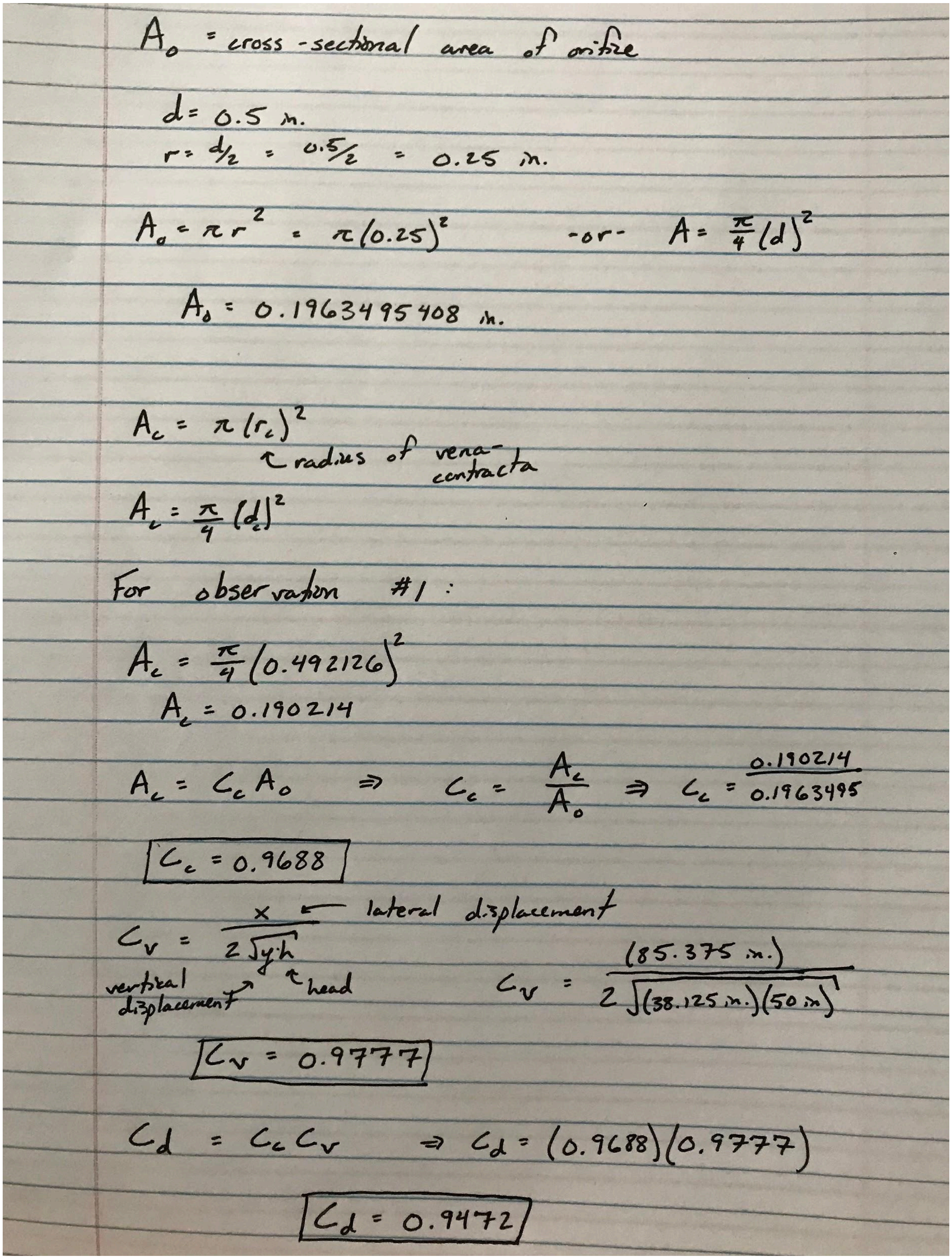
Experimental Set-Up and Procedures

Exercise 1: Determination of coefficient of contraction, velocity, and discharge.

We started by turning on the inflow valve and gradually opened the inlet valve to fill the tank, while keeping the orifice closed with a plug. Next once the flow depth in the tank had reached about 50 inches, we unplugged the orifice. Which will start draining the tank. We then opened the inlet valve further to adjust the inflow rate same as outflow rate to develop steady state in the tank. Next recording the "x" ordinates of the jet by using a sliding wire frame attached with the floor pan. We then recorded the “y” ordinate of the jet, which is the distance from the center of the orifice to the metal tape at the floor pan. Also noting that the value of “y” ordinate remains constant throughout the experiment. Next we measured the area of the free jet at the vena contracta using slide calipers. Then using the weighing bucket and the 2 inches plastic pipe links, we collected a known weight of water and recorded the time to collect the amount of water. We then changed the inflow rate to the tank to decrease the head by 5 inches and then adjusted the inflow and outflow rate in the tank. We repeated steps 3-6 to collect at least 6 sets of data by decreasing the head by 5 inch intervals.

Exercise 2: Time to empty the tank.

We started by plugging in the orifice meter. From there we opened the inlet valve and increased the head in the tank. When the head reached 50 inches, we turned off the inlet valve. We then unplugged the orifice meter and simultaneously recorded the time for the water level to drop from 50 inches to 45 inches to 40 inches to 35 inches and to 30 inches.



A_o = cross-sectional area of orifice

$d = 0.5$ in.

$r = d/2 = 0.5/2 = 0.25$ in.

$A_o = \pi r^2 = \pi(0.25)^2$ -or- $A = \frac{\pi}{4}(d)^2$

$A_o = 0.1963495408$ in.

$A_c = \pi(r_c)^2$

$\uparrow$ radius of vena-contracta

$A_c = \frac{\pi}{4}(d_c)^2$

For observation #1:

$A_c = \frac{\pi}{4}(0.492126)^2$

$A_c = 0.190214$

$A_c = C_c A_o \Rightarrow C_c = \frac{A_c}{A_o} \Rightarrow C_c = \frac{0.190214}{0.1963495}$

$\boxed{C_c = 0.9688}$

$C_v = \frac{x}{2\sqrt{y \cdot h}}$ ← lateral displacement

vertical displacement ↗ ↖ head

$C_v = \frac{(85.375 \text{ in.})}{2\sqrt{(38.125 \text{ in.})(50 \text{ in})}}$

$\boxed{C_v = 0.9777}$

$C_d = C_c C_v \Rightarrow C_d = (0.9688)(0.9777)$

$\boxed{C_d = 0.9472}$

Figure 7. Sample Calculations

Discussion

A_0 = 0.193495408 in.

Table 1. Data to Compute C_c , C_v , and C_d

# of Obs.	Head (in)	X (in)	Y (in)	Vena-Contracta (in)	Weight (lb)	Time (s)
1	50	85.375	38.125	0.492126	77	55.98
2	45	80.5	38.125	0.405512	50	50.18
3	40	75.75	38.125	0.413386	50	53.25
4	35	70.75	38.125	0.405512	50	57.42
5	30	65.875	38.125	0.409449	50	97
6	25	60.25	38.125	0.389764	50	66.01
7	20	52.375	38.125	0.374016	50	74.23
8	15	44	38.125	0.350394	50	85.41

Table 2. Time to Empty the Tank

Head Drop	55” to 50”	50” to 45”	45” to 40”	40” to 35”	35” to 30”
Time (s)	108	114	120	128	139

Table 3. Calculated Data

A_c	C_c	C_v	C_d
0.190214	0.9688	0.9777	0.9472
0.129151	0.6578	0.9718	0.6393
0.134215	0.6836	0.9699	0.6630
0.129151	0.6578	0.9684	0.6370
0.131671	0.6706	0.9739	0.6531

0.119315	0.6077	0.9758	0.5930
0.109867	0.5595	0.9484	0.5306
0.096428	0.4911	0.92	0.4518

1.The Coefficient of velocity ratio usually ranges from 0.96 to 0.99, and our experimental values between 0.92 to 0.9777.

2.The Coefficient of contraction is ideally 0.611 and less than 1. Our values range between 0.4911 to 0.9688, so we have one outlier.

3.The Coefficient of discharge displays the losses of energy in a system. It can be found by finding the direct measurement of flow rate and using Torricelli's Theorem.

While performing this experiment, there are multiple possibilities of potential errors that occurred. One of the possible errors is the wire measuring the displacement was not straight and had bends, so the accuracy of the reading could be skewed. Another is the diameter measurements of the vena contracta may not be too accurate due to the method of measuring. Lastly, our very last measurement for the 1st exercise was not as steady as our previous 7.

Conclusion

In this experiment, we performed eight tests and obtained data for vertical/horizontal displacements, vena contracta, and time needed for water to fill to 50lbs, while also utilizing Bernoulli's principle Using this data we were able to find flow rate, coefficients of contraction,velocity, and discharge across the orifice. By looking at the table, it can be observed that as the cross sectional area decreases, so do the coefficient values.

References

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